SIMULTANEOUS QUADRUPLE SERIES EQUATIONS INVOLVING HEAT POLYNOMIALS

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ABSTRACT

In the present paper an exact solution of the simultaneous quadruple series equations involving heat polynomials $P_{n,\upsilon}(x, t)$ is given. We have also shown the solution of the simultaneous quadruple series equations involving generalized Laguerre polynomials as a special case of the equations considered in the present paper.

KEYWORDS: Simultaneous quadruple series ,equations, polynomials

INTRODUCTION

In the present paper, we consider the following simultaneous quadruple series equations:

$$\sum_{n=o}^{(}\sum_{j=1}^{s} a_{ij}\frac{A_{nj}\, t^{-n_i}\ (^{-n_i}}{(\left((+\frac{1}{2}+n_i+p\right)}\quad \mathbf{nd\ using\ (2.3)to\ (2.5), we\ find} \qquad [1.1]$$

$$n = o \quad \sum\left(\sum_{j=1}^{s} b_{ij}\frac{A_{nj}}{(\left((+\frac{1}{2}+n_i+p\right)}\right)nd \quad \square\ \mathbf{using\ (2.3)to\ (2.5), we\ find} \qquad [1.2]$$

$$n = o \quad \sum\left(\sum_{j=1}^{s} b_{ij}\frac{A_{nj}}{(\left((+\frac{1}{2}+n_i+p\right)}\right)nd \quad \square\ \mathbf{using\ (2.3)to\ (2.5), we\ find} \qquad [1.3]$$

$$n = o \sum \left(\sum_{j=1}^{s} c_{ij} \frac{A_{nj}}{(((+ \frac{1}{2} + n_i + p)} \right) nd \quad \text{using } (2.3) \text{ to } (2.5), \text{ we find} \qquad [1.4]$$

Where i = 1,2,……………………,s.

Where, f_i (x, t), ϕ_i (x, t), Ψ_i (x, t) and g_i (x,t) are prescribed functions for $t \geq \rho > o$ and a_{ij}, b_{ij}, c_{ij} are known costants and sequence { A■(@n■(@j) })■(@) is to be determined.

)nd using (2.3)to (2.5), we find (x, t) is the heat polynomials (Haimo, 1966) defined by

$$P_1(n, \quad "(" \ ) (x, t) = \sum_1 (k = o)^{\dagger} n \ [\![2]\!]^{\dagger} 2k \ (\blacksquare(n@k)) \ (" (" ("(" + 1/2 + n) \ )/"(\qquad [1.5]$$

It may be noted that $P_{n,o}$ (x, t) = υ_{2n} (x,t) is the ordinary heat polynomial of even order defined by Rosenbloom and Widder (1959) and that

$P_{n,o}$ (x, -1) = $(-1)^n$. 2^{2n} (n)! $L_n^{-1/2}\left(\frac{x^2}{4}\right)$ = H_{2n} $\left(\frac{x}{2}\right)$, the Hermite polynomial of even order defined by Erdelyi (1953).

We also define

$$W_{n,\upsilon}(x,t) = t^{-2n} G_{\upsilon}(x,t)\, P_{n,\upsilon}(x,-t),\ t > 0 \qquad [1.6]$$

$$\text{Where, } G_{\upsilon}(x,t) = (2t)^{-\upsilon-1/2} \exp.(-x^2/4t), \qquad [1.7]$$

and $W_{n,\upsilon}(x,t)$ is the Appell transform of $P_{n,\upsilon}(x, -t)$.

The analysis is purely formal and no attempt is made to supply details of rigours proof.

CERTAIN INTEGRAL AND SERIES REPRESENTATIONS

The heat polynomial $P_{n,\upsilon}(x,t)$ is related to the generalized Laguerre polynomial by

$$P_{n,\upsilon}(x,-t) = (-1)^n\ 2^{2n}\ (n)!\ t^n\ L_n^{(\upsilon-\frac{1}{2})}\left(\frac{x^2}{4t}\right), \qquad [1.8]$$

Using the orthogonality relation for $L_n^{(\alpha)}$ (x), it can easily be verified that for t>o,

$$\int_0^\infty w_{m,\upsilon}(x,t)\, P_{n,\upsilon}(x,-t)\, d\,R(x) = \delta_{m,n}\, k_n.$$

[1.9]

Where

$$d\,R(x) = 2^{1/2-\upsilon}\,[\,\Gamma(\upsilon+1/2)]^{-1}\,.\,x^{2\upsilon}\,dx \qquad [1.10]$$

and

$$K_n = \frac{\Gamma\left(\upsilon+\frac{1}{2}\right)}{\left[2^{4n}(n)!\,\Gamma\left(\upsilon+\frac{1}{2}+n\right)\right]} \qquad [1.11]$$

Now, using the formula (27), pp. 190 of Erdelyi[1] in the form

$$\left(\frac{d}{dx}\right)^m \{x^{\alpha+m}\,L_n^{(\alpha+m)}(x)\} = \frac{\Gamma(\alpha+m+n+1)}{\Gamma(\alpha+n+1)}\ x^{\alpha}\,L_n^{(\alpha)}(x), \qquad [1.12]$$

and from the relation [1.8], we obtain at once that

$$\left(\frac{d^2}{dx^2}\right)^m \left\{x^{2\upsilon+2m-1}\,P_{n,\upsilon+m}(x,-t)\right\}$$

$$= \frac{\Gamma\left(\alpha + \frac{1}{2} + m + n\right)}{\Gamma\left(\alpha + \frac{1}{2} + n\right)} \left(x^{2\alpha-1}\right) P_{n,\alpha}(x, -t)$$ [1.13]

The relation

$$[e]^{\dagger}(\mathrm{I}x)\, L_{\mathrm{I}} n^{\dagger}((\alpha))\, (x) = [(-1)]^{\dagger} m\, [(d/dx)]^{\dagger} m \{ [e]^{\dagger}(-x)\, L_{\mathrm{I}} n^{\dagger}((\alpha$$ [1.14]

together with [1.8] yields

$$(-4t)^{m} \left(\frac{d^2}{dx^2}\right)^{m} \left\{ e^{\frac{-x^2}{4t}} P_{n,\alpha}(x, -t) \right\} = e^{\left[-\frac{(x)^2}{4t}\right)} p_{n,\alpha}(x, -t)$$ [1.15]

Now, we derive a few fractional integral type representations for

$P_{n,\alpha}$ (x,-t) and $w_{n,\upsilon}(x,t)$. Using the definition of Beta function and integrating the series for $P_{n,\upsilon}(x,-t)$ term by term with respect to x, it can easily be seen that

$$P_{\blacksquare}(@n, "(" + \beta)(\xi, -t) = 2\xi_{\blacksquare}(-2" (" - 2\beta + 1@)\ ("\Gamma(\beta + " (" + 1/2 " + n)")/\Gamma(\beta)\Gamma("(" + 1/2 + n)$$

$$(\beta > 0, (> -1/2)$$

[1.16]

Using the following form of the Beta function formula

$$= \frac{\Gamma(\lambda - \mu)\, \Gamma(\mu + s)}{\Gamma(\lambda + s)} \left(t^{-s-\lambda}\right)$$ [1.17]

where $\lambda > \mu$ and $s+\mu>0$ and integrating the series for $p_{n,\alpha}(x, -t)$ term by term with respect to t , we get

$$\int_{(}^{(} t^{(-n-(-1/2)} (t-()^{((-(-1)} p_{n,(}(x,-t))\, dt =$$

$$(((((-( )" ((((+1/2+n))/"((((+1/2+n) \; (^{(-(((+1/2+n))} P_{(n} \qquad [1.18]$$

Expressing $P_{n,\upsilon}(x,-t)$ in terms of generalized Lagurerre polynomial by means of [1.8] and using the formula of Erdelyi[2] pp. 403.

$$\int_{(}^{(} e^{(-x)} (x-( )^{((-1)} \; L_{n}^{((\alpha))}(x)\,dx = ((() \; e^{(-( )} L_{n}^{((\alpha} \qquad [1.19]$$

It can be proved that

$$P_{(}(n,((-( ) \; (((,-t) = 2^{(1-2(( )} \; t^{(-( )/(((} \; e^{((((^{2}/4t))} \int_{(}^{(} x(x^{2}-(^{2}$$

$$((+1/2>(>0)$$

Now, we derive certain series representation for $P_{n,\upsilon}(x,t)$. Using the generating relation to Haimo (1966).

$$(1-4zt)^{(-((-1/2)} \; e(((x(2@)@)z)/((1-4zt))@) = \sum_{(n=0)}^{(} (z^{n})/((n)!) \; p$$

and the relation

$$(1-4zt)^{-v-\frac{1}{2}} e^{\frac{x^2 z}{1-4zt}} = (1-4zt)^{-(\upsilon-\mu)} (1-4zt)^{-\frac{1}{2}-\mu} e^{\frac{x^2 z}{1-4zt}}$$

It follows that

$$P_{n,(}(x,t) = \frac{((n+1)}{(((-()} \sum_{k=0}^{n} \frac{(((-(+n-k)\,(4t)^{n-k}}{((n-k+1)\,((k+1)} \cdot p_{k,(}(x,t), \qquad [1.22]$$

Equation [1.22] can be inverted to get

$2^{(1/2 - "(")}$ 〚 x〛 $^{2(}$ 〚 t〛 $^{(-2k)}$ 〚 G〛 $_{(}$ 〚(x, t) "(" 〛 $_{(k, "("}$) (x, −t)

$= \Sigma_{(n = k)}^{\infty}$ 〚 ("(" ("(" − "(" + n − k) "(" ("(" + 1/2 + k)))/("(" ("(" − "(")"(

〚 (−t/4)〛 $^{(n-k)}$ $2^{(1/2 - "(")}$ 〚 x〛 $^{2(}$ 〚 t〛 $^{(-2n)}$ 〚 〚 G〛 $_{("("}$) (x, t) ' [1.23]

$(2\mu > \upsilon)$

Equation [1.23] follows from [1.22], since they each are equivalent to

$\int_0^{\infty} 2^{(1/2 - "(")}$ 〚 x〛 $^{2(}$ 〚 t〛 $^{(-2k)}$ 〚 G〛 $_{("("}$) (x, t) 〚 〚 "(" 〛 $_{(k, "(}$

= ("(" ("(" − "(" + n − k) "(" ("(" + 1/2 + k)))/"(" ("(" − "(")" (" (n − k + 1) (− [1.24]

SOLUTION OF QUADRUPLE SERIES EQUATIONS

Multiplying equations [1.1] by $(t)^{-\left(p+m+(+\frac{1}{2}\right)}(t-\alpha)^{(+m-(-1}$, where m is a positive integer, integrating with respect to t from p to ∞ and using [1.18], we get

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij} \frac{A_{nj}}{(\left((+m+ni+p+\frac{1}{2}\right)} \quad (_{ni+p,(+m} \; ((,-() = F_i((,() \qquad [2.1]$$

where $0 < (< y$,

where $\mu + m > \upsilon > -1/2$ and

$F_i(@i)((,() - (^{("("} + 1/2)/"(" ("(" + m - "(") \int_{(}^{\infty}$ 〚 $t^{(-(p + m + "(" + 1/2))}$ (t −

[2.2]

Further, setting t = ρ in [1.4], multiplying it by x

〖(x↑2 − (↑2)〗↑(″(″ − ″(″ − 1) e↑(〖 − x 〗↑2/4) , integrating with respect to x from ξ to

∞ and applying [1.20], we obtain

Σ↓(n = 0)↑∞▒ 〖Σ↓(j = 1)↑s▒ 〖b■(@i■(@j))〗 (A↓(n■(@j)))/("(" ("(" + ni + p + 1/2))〗 〖 "(" 〗↓(ni

Where e^{ij} are the elements of the matrix $[b_{ij}][c_{ij}]^{-1}$.

where $s > m > -\frac{1}{2}$ and

G■(@i)("(" ,"(") = 2↑(1 − 2("(" − "(")) (↑(−("(" − "(")) e↑(〖("(" 〗↑2/4" ("))/"(" ("

i = 1,2,.................,s . [2.4]

Now, multiplying [2.1] by (↑(2("(" + m) − 1) and applying the operator $\frac{d^2}{(d(\square)^2)^m}$

,

We see in view of [1.13] that

Σ↓(n = 0)↑∞▒ 〖Σ↓(j = 1)↑s▒ 〖b■(@i■(@j))〗 (A↓(n■(@j)))/"(" ("(" + 1, [2.5]

Where

H■(@i)("(" ,"(") = (↑(1 − 2" (") ((d↑2)/ 〖d(〗↑2)↑m [(↑(2("(" + m) [2.6] and d^{ij} are the elements of the matrix $[b_{ij}][a_{ij}]^{-1}$

The left hand sides of [2.5] , [1.2] , [1.3] and [2.3] are now identical and an application of the orthogonality relation [1.9] yields the solution of the equations [1.1] , [1.2] , [1.3] and [1.4] in the form:

A₁(n∎(@j)) = Σ₁(j = 1)^s ⟦f∎(@i∎(@j))⟧ (" (" ("(" + 1/2))/(2^4(n∎(@i) + p) (n∎(@

$$\sum_{j=1}^{s} e_{ij} + h \int^{\left(W_{(n_1+p,\ldots)}\right]}$$

[2.7]

where, H_i (ξ,ρ) and G_i (ξ,ρ) are the same as defined by [2.6] and [2.4] respectively and $dR(\xi)$ is defined by [1.10] and f^{ij} are the elements of the matrix $[b_{ij}]^{-1}$.

Using the relation [1.8] and setting

$B_n = A_n (-1)^{n+p} 2^{2(n+p)}(n+p)!$, we find that the equations [1.1], [1.2], [1.3] and [1.4] transform into

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij} \frac{B_{nj} t^{ni}}{\left(\left(\left(+\frac{1}{2}+ni+p\right)\right.\right.} L_{ni+p}^{\left(\left(-\frac{1}{2}\right)\right.}\left(\frac{x^2}{4t}\right) = t^{-(} f_i(x,t), \qquad 0 < x < y, \qquad [2.8]$$

Σ↓(n = 0)^∞ ⟦Σ₁(j = 1)^s ⟦b∎(@i∎(@j))⟧ (B₁(n∎(@j)) ⟦ t⟧^ni)/"(" ("(" + 1 [2.9]

Σ↓(n = 0)↑∞▒ 〖Σ↓(j = 1)↑s▒ 〖b■(@i■(@j))〗 (B↓(n■(@j)) 〖 t〗↑ni)/"(" ("(" + : [2.10]

Σ↓(n = 0)↑∞▒ 〖Σ↓(j = 1)↑s▒ 〖c■(@i■(@j))〗 (B↓(n■(@j)) 〖 t〗↑ni)/"(" ("(" + 1 [2.11]

Where, t ((> 0 , and their solution is given by

(² / 4(⬚)H↓j((, ()dR((⬚)

+∫↓y↑z▒e↑(〖 − ("(" 〗↑2/4" (")) 〖L↓(ni + p)↑(("(" − 1/2)) ((↑2/4()〗↑ (■(@i)(

+∫↓z↑h▒ 〖e↑(〖 − ("(" 〗↑2/4" (")) L↓(ni + p)↑(("(" − 1/2)) ((↑2/4() 〗 (■(@i)("(" ,"(")dR(()

+ ∑↓(j = 1)↑s▒ 〖e■(@i■(@j))〗 ∫↓h↑(▒ 〖e↑(〖 − ("(" 〗↑2/4" (")) L↓(ni + p)↑(

[2.12]

Where H**i** (ξ,ρ) , G**i** (ξ,ρ) and dR (ξ) are the same as defined by[2.6], [2.4] and [1.10].

The solution of simultaneous quadruple equations involving generalized Languerre polynomials can be obtained independently by the above procedure.

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